First Semester B.Sc. Degree Examination, October/November 2019

(CBCS Scheme)

Mathematics

Paper 1.1 - ALGEBRA AND CALCULUS - I

Time: 3 Hours

[Max. Marks: 90

Instructions to Candidates:

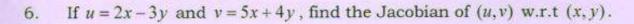
- 1. Answer ALL Questions.
- 2. Answer should be written completely in English.

PART - A

Answer any SIX of the following:

 $(6\times 2=12)$

- 1. Find the nth derivative of $\cos^2 x$.
- 2. Define the point of inflexion.
- 3. For the curve $x = a\cos t$ and $y = b\sin t$. Find $\frac{ds}{dt}$.
- 4. Evaluate $\int_0^{\pi/2} \cos^8 x dx$.
- 5. If $u = \log(x^2 + y^2 + z^2)$ find $\frac{\partial u}{\partial x}$ and $\frac{\partial u}{\partial y}$.



7. Find the rank of the matrix
$$A = \begin{bmatrix} 1 & 2 & -1 & 3 \\ 2 & 4 & -4 & 7 \\ -1 & -2 & -1 & 2 \end{bmatrix}$$

 Prove that eigen vector of a matrix corresponds to one and only one eigen value of A.

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PART - B

Answer any SIX of the following:

 $(6 \times 3 = 18)$

- 9. Find the nth derivative of $\frac{1}{x^2 + 3x 10}$.
- 10. Find the length of perpendicular from pole to the tangent for the curve $r = a(1 + \cos \theta)$.
- 11. Find $\frac{ds}{dx}$ for the curve $x = a(1 \cos \theta)$ and $y = a(\theta + \sin \theta)$.
- 12. Evaluate $\int_{0}^{\pi} x \sin^5 x \, dx$
- 13. If $u = \tan^{-1}(y/x)$ prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$.
- 14. If $u = \sin^{-1}(x y)$, x = 3t and $y = 4t^3$ then show that $\frac{du}{dt} = \frac{3}{\sqrt{1 t^2}}$.
- 15. Find the value of 'a' using row reduced echelon form where $A = \begin{bmatrix} 6 & a & -1 \\ 2 & 3 & 1 \\ 3 & 4 & 2 \end{bmatrix}$ has rank 2.
- 16. Find the eigen values of the matrix $A = \begin{bmatrix} 3 & 2 & -2 \\ -3 & -1 & 3 \\ 1 & 2 & 0 \end{bmatrix}$.

PART- C

Answer any THREE of the following :

 $(3 \times 5 = 15)$

- 17. Find the nth derivative of $e^{ax} \cdot \cos(bx + c)$.
- 18. If $y = e^{m \sin^{-1} x}$ prove that $(1 x^2) y_{n+2} (2n+1) x y_{n+1} (n^2 + m^2) y_n = 0$.

- 19. With usual notations prove that $\tan \phi = r \frac{d\theta}{dr}$ for the curve $r = f(\theta)$.
- 20. Find the pedal equation of the curve $x^2 + y^2 = 2ax$.

PART- D

Answer any THREE of the following:

 $(3 \times 5 = 15)$

- 21. Find the radius of curvature at any point for the curve $x = a\cos^3 t$, $y = a\sin^3 t$.
- 22. Show that the evaluate of the curve $x = a(\cos t + t \sin t)$ and $y = a(\sin t t \cos t)$ is $x^2 + y^2 = a^2$.
- 23. Find all the asymptotes of the curve $x^3 + x^2y xy^2 y^3 3x y 1 = 0$.
- 24. Obtain the reduction formula for $\int_{0}^{\pi/2} \cos^{n} x dx$.

PART - E

Answer any THREE of the following :

 $(3 \times 5 = 15)$

- State and prove Euler's theorem on homogeneous function.
- 26. If $u = x^2 2y$, v = x + y, then find $J = \frac{\partial(u, v)}{\partial(x, y)}$ and $J' = \frac{\partial(x, y)}{\partial(u, v)}$ also verify that $J \cdot J' = 1$.
- 27. Expand $e^x \cos y$ using Taylor's theorem at $(1, \pi/4)$ upto second degree terms.
- 28. Find the extreme value of $f(x,y) = x^3 + 3xy^2 15x^2 15y^2 + 72x$.

PART - F

Answer any THREE of the following:

 $(3\times 5=15)$

29. Solve completely the system of equations

$$x_1 + 2x_2 + 2x_3 = 0$$

$$2x_1 + x_2 + x_3 = 0$$

$$3x_1 + 2x_2 + 2x_3 = 0$$

$$x_2 + x_3 = 0$$



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30. For what values of M and n the system of equations

$$x+2y+3z=4$$

$$x+3y+4z=5 \text{ has}$$

$$x+3y+mz=n$$

- (a) unique solution
- (b) no solution
- (c) infinite number of solutions.
- 31. Diagonalize the matrix $\begin{bmatrix} 7 & 3 \\ 3 & -1 \end{bmatrix}$.
- 32. Verify the Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}$ and hence find A^{-1} .