First Semester B.Sc. Degree Examination, October/November 2019

(CBCS Scheme)

Mathematics

Paper 1.1 - ALGEBRA AND CALCULUS – I

Time : 3 Hours] *[Max. Marks : 90*

Instructions to Candidates :

1. *Answer ALL Questions.*
2. *Answer should be written completely in English.*

PART – A

Answer any **SIX** of the following : **(6 × 2 = 12)**

1. Find the nth derivative of $\cos^2 x$.

2. Define the point of inflexion.

3. For the curve $x = a\cos t$ and $y = b\sin t$. Find $\frac{ds}{dt}$.

4. Evaluate $\int_0^{\pi/2} \cos^8 x dx$.

5. If $u = \log(x^2 + y^2 + z^2)$ find $\frac{\partial u}{\partial x}$ and $\frac{\partial u}{\partial y}$.

6. If $u = 2x - 3y$ and $v = 5x + 4y$, find the Jacobian of (u, v) w.r.t (x, y).

7. Find the rank of the matrix $A = \begin{bmatrix} 1 & 2 & -1 & 3 \\ 2 & 4 & -4 & 7 \\ -1 & -2 & -1 & 2 \end{bmatrix}$

8. Prove that eigen vector of a matrix corresponds to one and only one eigen value of A.

PART – B

Answer any **SIX** of the following : **(6 × 3 = 18)**

9. Find the nth derivative of $\frac{1}{x^2+3x-10}$.

10. Find the length of perpendicular from pole to the tangent for the curve $r = a(1+\cos\theta)$.

11. Find $\frac{ds}{dx}$ for the curve $x = a(1-\cos\theta)$ and $y = a(\theta + \sin\theta)$.

12. Evaluate $\int_0^{\pi} x\sin^5 x\, dx$.

13. If $u = \tan^{-1}(y/x)$ prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$.

14. If $u = \sin^{-1}(x-y)$, $x = 3t$ and $y = 4t^3$ then show that $\frac{du}{dt} = \frac{3}{\sqrt{1-t^2}}$.

15. Find the value of 'a' using row reduced echelon form where $A = \begin{bmatrix} 6 & a & -1 \\ 2 & 3 & 1 \\ 3 & 4 & 2 \end{bmatrix}$ has rank 2.

16. Find the eigen values of the matrix $A = \begin{bmatrix} 3 & 2 & -2 \\ -3 & -1 & 3 \\ 1 & 2 & 0 \end{bmatrix}$.

PART– C

Answer any **THREE** of the following : **(3 × 5 = 15)**

17. Find the nth derivative of $e^{ax} \cdot \cos(bx+c)$.

18. If $y = e^{m\sin^{-1}x}$ prove that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+m^2)y_n = 0$.

19. With usual notations prove that $\tan\phi = r\dfrac{d\theta}{dr}$ for the curve $r = f(\theta)$.

20. Find the pedal equation of the curve $x^2 + y^2 = 2ax$.

PART- D

Answer any **THREE** of the following : **(3 × 5 = 15)**

21. Find the radius of curvature at any point for the curve $x = a\cos^3 t$, $y = a\sin^3 t$.

22. Show that the evaluate of the curve $x = a(\cos t + t\sin t)$ and $y = a(\sin t - t\cos t)$ is $x^2 + y^2 = a^2$.

23. Find all the asymptotes of the curve $x^3 + x^2y - xy^2 - y^3 - 3x - y - 1 = 0$.

24. Obtain the reduction formula for $\int_0^{\pi/2}\cos^n x\,dx$.

PART - E

Answer any **THREE** of the following : **(3 × 5 = 15)**

25. State and prove Euler's theorem on homogeneous function.

26. If $u = x^2 - 2y$, $v = x + y$, then find $J = \dfrac{\partial(u,v)}{\partial(x,y)}$ and $J' = \dfrac{\partial(x,y)}{\partial(u,v)}$ also verify that $J \cdot J' = 1$.

27. Expand $e^x \cos y$ using Taylor's theorem at $(1, \pi/4)$ upto second degree terms.

28. Find the extreme value of $f(x,y) = x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$.

PART - F

Answer any **THREE** of the following : **(3 × 5 = 15)**

29. Solve completely the system of equations

$$x_1 + 2x_2 + 2x_3 = 0$$
$$2x_1 + x_2 + x_3 = 0$$
$$3x_1 + 2x_2 + 2x_3 = 0$$
$$x_2 + x_3 = 0$$

30. For what values of M and n the system of equations

$$x+2y+3z=4$$
$$x+3y+4z=5 \text{ has}$$
$$x+3y+mz=n$$

(a) unique solution

(b) no solution

(c) infinite number of solutions.

31. Diagonalize the matrix $\begin{bmatrix} 7 & 3 \\ 3 & -1 \end{bmatrix}$.

32. Verify the Cayley-Hamilton theorem for the matrix $A=\begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}$ and hence find A^{-1}.

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